# Shift register with feedback generates white noise

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A shift register with linear feedback generates a pseudorandom sequence of pulses that can be used without digital-to-analog conversion or audio processing as extremely high-quality audio white noise. The output from the register, fed directly to an audio amplifier, produces a power spectrum that is flat to within ±1 decibel over the entire audio range.

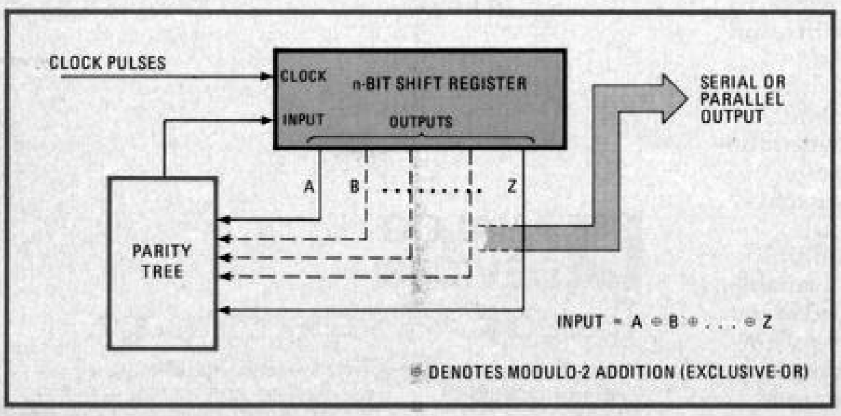
The operating principles of a linear-feedback shift register (LFSR) are illustrated in Fig. 1. The input to the first stage of an n-bit register is determined at each clock pulse by the exclusive-OR (parity) function of some output taps of the register. Choosing these taps is the crucial step in constructing a LFSR that performs as required.

For an n-bit shift register, taps can be chosen so that the register cycles through $2^n - 1$ different states before repeating any previous state. All possible n-bit words are generated except the word containing only 0s [*Electronics*, Nov. 27, 1975, p. 104]. In addition, with the use of only two taps, some shift-register lengths can produce these maximal-length sequences. A partial list of such registers is given in the table, which is excerpted from "Shift Register Sequences," by S. Golomb (Holden-Day Inc., San Francisco, 1967). As the table shows, even shift registers that are only moderately long can produce astronomically long sequences.

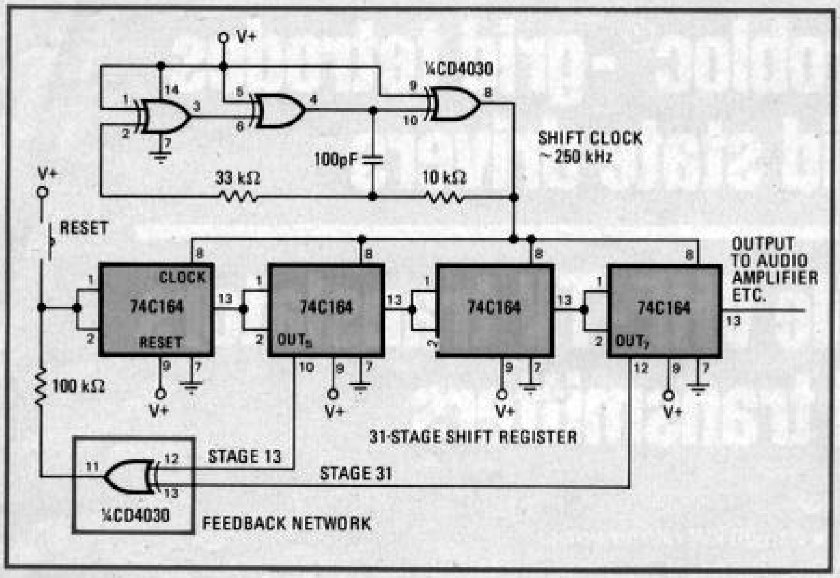
An appropriate clock and a sufficiently long register generate a flat power spectrum of audio white noise, using the digital bit stream itself as the noise source. Fig-

MAXIMUM-LENGTH LINEAR-FEEDBACK SHIFT REGISTERS THAT REQUIRE ONLY TWO FEEDBACK TAPS

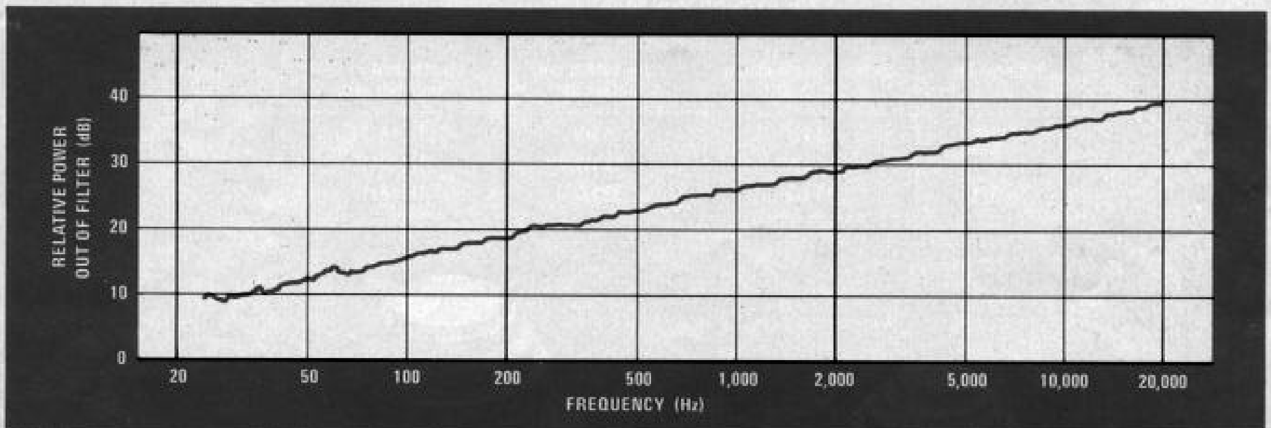
| No. of stages | Stages at which taps are placed | Sequence length | Duration of sequence using 250-kHz clock |
|---|---|---|---|
| 7 | 1, 7 or 3, 7 | 127 | 0.51 ms |
| 9 | 4, 9 | 511 | 2.0 ms |
| 10 | 3, 10 | 1,023 | 4.1 ms |
| 11 | 2, 11 | 2,047 | 8.2 ms |
| 15 | 1, 15 or 4, 15 or 7, 15 | 32,767 | 131 ms |
| 17 | 3, 17 or 5, 17 or 6, 17 | 131,071 | 0.52 s |
| 18 | 7, 18 | 262,143 | 1.0 s |
| 20 | 3, 20 | 1,048,575 | 4.2 s |
| 21 | 2, 21 | 2,097,151 | 8.4 s |
| 22 | 1, 22 | 4,194,303 | 17 s |
| 23 | 5, 23 or 9, 23 | 8,388,607 | 34 s |
| 25 | 3, 25 or 7, 25 | 33,554,431 | 2.2 m |
| 28 | 3, 28 or 9, 28 or 13, 28 | 268,435,455 | 18 m |
| 29 | 2, 29 | 536,870,911 | 36 m |
| 31 | 3, 31 or 6, 31 or 7, 31 or 13, 31 | 2,147,483,647 | 2.4 h |
| 33 | 13, 33 | 8,589,934,591 | 9.5 h |
| 35 | 2, 35 | 34,359,738,367 | 1.6 d |
| 36 | 11, 36 | 68,719,476,735 | 3.2 d |
| 39 | 4, 39 or 8, 39 or 14, 39 | $5.5 \times 10^{11}$ | 25 d |
| 41 | 3, 41 or 20, 41 | $2.2 \times 10^{12}$ | 102 d |




**1. Pseudorandom pulses . . .** In this linear-feedback shift register, some of the output ports are connected back to the input through an exclusive-OR circuit. Depending upon which output taps are fed back, a nonrepeating sequence of any length up to $2^n - 1$ binary words can be generated.




**2. . . . generate noise . . .** This 31-stage linear-feedback shift register is arranged to produce a maximum-length pseudorandom bit sequence by connection of stages 13 and 31 back to input. Output bit stream, which can be taken from any port, constitutes a white-noise source.




**3. . . . like this.** The output power spectrum of the circuit in Fig. 2, measured directly at the output of stage 31, slopes upward because filter bandwidth is proportional to frequency. The slope of 3 dB/octave indicates white noise. Reference level (0 dB) was chosen arbitrarily.

ure 2 shows a 31-stage LFSR, with taps at stages 13 and 31 and a shift clock running at 250 kilohertz.

Any shift register that provides access to the required feedback bits will serve. For instance, two CD4006s might have been used instead of the 74C164s. With only three ICs, these shift registers can give access to bits 13 and 31. For a white-noise generator in audio applications, the component values are noncritical. The reset button ensures that at least a single 1 is initially in the shift register, but the manual button can be replaced by a more elaborate initialization circuit if desirable.

The audio-power spectrum from the circuit in Fig. 2, measured directly at the output of stage 31, is shown in Fig. 3. A series of ⅓-octave filters measures the spectrum. The curve is inclined upward at a rate of 3 decibels per octave, matching the increasing bandwidth of the filters. The deviation from a straight line inclined 3 dB/octave is less than 1 dB over the frequency interval from 25 Hz to 20 kHz. The largest deviation occurs at the power-line frequency of 60 Hz. The table shows that the string produced by this register is longer than 2 billion bits and, at a 250-kHz clock rate, will take more than two hours to repeat.

The LFSR pulse sequences are also used for error-correcting codes, spread-spectrum techniques [*Electronics*, May 29, 1975, p. 127], and other random-selection processes. In a maximum-length LFSR n bits long, the bit string produced is statistically identical to $2^n - 1$ flips of an ideal coin (one with precisely equal probabilities of landing heads or tails). Thus, for example, a 17-stage LFSR can generate the equivalent of 131,071 coin-flips. Any stage of the register may provide the output, since every bit is eventually shifted the entire length of the register.

Such a device could be useful for producing uncorrelated stimuli in a psychophysical experiment, because it could easily determine which of two possible stimuli to present to a test subject. It can do so with an undiscernible, yet repeatable, pattern so that a second test subject could be given the same sequence of stimuli. If the bit string from the 31-stage register in Fig. 2 were used for test stimuli with an average interval between stimuli of 5 seconds, it would not repeat for 340 years. □

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